CHARLES DARWIN UNIVERSITY

Family Name	
Given Names	
Student Number	
Teaching Period	Semester 1, 2016

FINAL EXAMINATION	**DURATION**
ENG325 – Systems Modelling and Control	Reading Time: **10** minutes Writing Time: **180** minutes

INSTRUCTIONS TO CANDIDATES

1.1 The examination has five (5) questions.

1.2 All answers must include a sufficient amount of working and explanation.

1.3 Note that questions ARE NOT of equal value.

1.4 Read ALL questions carefully.

EXAM CONDITIONS

You may begin writing from the commencement of the examination session. The reading time indicated above is provided as a guide only.

This is a CLOSED BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

No dictionaries are permitted

ADDITIONAL AUTHORISED MATERIALS	**EXAMINATION MATERIALS TO BE SUPPLIED**
No additional printed material is permitted	1 x 20 Page Book 2 x Scrap Paper Formula Sheet/s

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

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Question 1 (13 marks)

Question 1.1 (8 Marks)

For each of the following terms, give a brief definition as they relate to continuous and discrete time systems.

(a) Causal
(b) Memory less
(c) Time invariant
(d) Linear

Question 1.2 (2 Marks)

What is the definition of the impulse response, h(t), of a system? How can it be used to find the output of a system for a given input?

Question 1.3 (3 Marks)

What is the drawback of trying to determine the impulse response of a system experimentally? How and why can the unit step function be utilised instead?

Question 2 (12 marks)

A continuous-time periodic signal x(t) is real valued and has a fundamental period
T = 4 seconds.

The nonzero Fourier series coefficients for x(t) are;

$C_1 = C_{-1} = 3$;
$C_3 = C^*_{-3} = 5j$, that is C_{-3} is the complex conjugate of C_3

Question 2.1 (3 Marks)

Sketch two graphs, one showing the magnitude of x(t) vs. k, the other phase vs. k from k = -5 to 5.

Question 2.2 (4 Marks)

Express x(t) in the form;

$$x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k)$$

Question 2.3 (3 Marks)

An ideal high pass filter is then used on the signal x(t) to pass only the higher frequency component and block the low frequency component.

If this filter has a magnitude of 1 and a phase of 2ω in the pass band and a phase and magnitude of zero in the stop band, write the expression for the output of the filter, v(t).

Question 2.4 (2 Marks)

What are the drawbacks of the type of filter used in question 2.3. Under what circumstances can we use this kind of filter to extract the high frequencies from a signal?

Question 3 (10 marks)

A signal *x(t)* has the frequency spectrum shown in Figure 3a, where A = 1 and B = 5. The frequency, ω, is measured in radians per second.

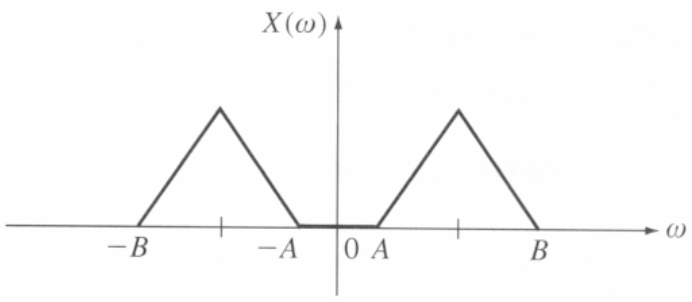

Figure 3a, Frequency Spectrum

Question 3.1 (2 marks)

The signal is multiplied with a carrier $\cos\omega_c t$. Sketch the amplitude spectrum $S(\omega)$ of the modulated carrier $s(t) = x(t)\cos\omega_c t$, for $\omega_c = 15$.

Question 3.2 (3 marks)

The modulated carrier *s(t)* is filtered. The frequency function of the filter is given by

$$|H(\omega)| = \begin{cases} 2, & 10 \le |\omega| \le 20 \\ 0, & \text{all other } \omega \end{cases}$$

Sketch the amplitude spectrum $V(\omega)$ of the filtered signal *v(t)*. Can the filter described above be used in real time?

Question 3.3 (1 marks)

The signal *v(t)* is transmitted. What, specifically, is this form of transmission called? Do not use abbreviations.

Question 3.4 (4 marks)

Explain in detail (include diagrams and formula if necessary) how the original signal *x(t)* can be reconstructed. Can the signal be reconstructed if $\omega_c = 3$, instead of 15?

Question 4 (15 marks)

One of the research strengths of Charles Darwin University is modeling of Darwin harbour. One of the main topics of investigation is the effect of the location of the sewage outlet on the levels of E. Coli (bacteria that are usually present in the gut) in Darwin harbour. So far, the effect of the current outlet of untreated sewage at Larrakeyah has been modeled. It is now proposed to change the location of the outlet to Rapid Creek. As a first attempt the following model is proposed:

$$\frac{dq(t)}{dt} = -k_1 q(t) + x(t) \quad and \quad \frac{dy(t)}{dt} = k_1 q(t) - k_2 y(t)$$

where the input $x(t)$ is the rate at which sewage (and therefore E. Coli) flows into Rapid Creek, the output $y(t)$ is the total number of E. Coli in Darwin Harbour and $q(t)$ is the total number of E. Coli in Rapid Creek. The constants k_1 and k_2 characterize the speed with which sewage flows from Rapid Creek into the Darwin Harbour and the speed at which bacteria disappear from the harbour (either because they die or because they flow into the sea) respectively. It is assumed that $k_1 > k_2 > 0$.

Question 4.1 (5 marks)
Determine the transfer function of the system described by the equations above and the input/output differential equation of the complete system. For this question only, it is ok to assume that the initial conditions for $q(t)$ and $y(t)$are zero.

If and only if you cannot find the transfer function, use the following transfer function for question 4.2 – 4.6

$$H(s) = \frac{k_1 k_2}{(s+k_1)(s+k_2)}$$

Question 4.2 (1 marks)
What is the order of the system?

Question 4.3 (1 marks)
What is the location of the poles of the system?

Question 4.4 (2 marks)
Is the system stable? Explain your answer.

Question 4.5 (2 marks)
Determine the impulse response *h(t)* of the system.

Question 4.6 (4 marks)
It is proposed that in 2018 a new sewage treatment plant will be opened and the outflow of untreated sewage will be stopped. Assume that treated sewage does not contain any E. Coli, in other word the release of E. Coli is stopped completely. Assume that at that time (*t=0*) the number of E. Coli in Rapid Creek is EC_1 and the number of E. Coli in Darwin Harbour is EC_2. By using the Laplace transform, compute the number of E. Coli in Darwin Harbour, *y(t),* from this time onward. Assume that $k_1 \neq k_2$.

Question 5 (15 marks)

Question 5.1 (2 marks)

A controller with transfer function $G_C(s)$ is used to provide the input into a plant with transfer function $G_P(s)$. The input to the controlled system is the reference signal $R(s)$which produces the output, $Y(s)$, from the plant. Draw the block diagram for the system described if:

a) open loop control is used
b) closed loop control is used

Question 5.2 (2 marks)

Give an example of an everyday system where open loop control is used and another where closed loop control is used.

Question 5.3 (5 marks)

What are the reasons that closed loop control is sometimes used instead of open loop control. What are some of the advantages and disadvantages of closed loop control?

Question 5.4 (6 marks)

Discuss, with the use of formula and diagrams where relevant, how you would use one of the two tuning methods discussed in this unit to tune a PI controller.

Formula Sheet

LAPLACE TRANSFORM PAIRS

F(s)	f(t) $t \ge 0$
1	$\delta(t)$ unit impulse at t = 0
$\frac{1}{s}$	1 or $u(t)$ unit step starting at t = 0
$\frac{1}{s^2}$	$tu(t)$ ramp function
$\frac{1}{s^n}$	$\frac{1}{(n-1)!}t^{n-1}$ n = positive integer
$\frac{1}{s}e^{-at}$	$u\ (t-a)$ unit step starting at t = a
$\frac{1}{s}\left(1-e^{-at}\right)$	$u\ (t)-u\ (t-a)$ rectangular pulse
$\frac{1}{s+a}$	e^{-at} exponential decay
$\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$ n = positive integer
$\frac{1}{s(s+a)}$	$\frac{1}{a}\left(1-e^{-at}\right)$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}\left[1-\frac{b}{b-a}e^{-at}+\frac{a}{b-a}e^{-bt}\right]$
$\frac{s+\alpha}{s(s+a)(s+b)}$	$\frac{1}{ab}\left[\alpha-\frac{b(\alpha-a)}{b-a}e^{-at}+\frac{a(\alpha-b)}{b-a}e^{-bt}\right]$
$\frac{1}{(s+a)(s+b)}$	$\frac{1}{b-a}\left(e^{-at}-e^{-bt}\right)$
$\frac{s}{(s+a)(s+b)}$	$\frac{1}{a-b}\left(ae^{-at}-be^{-bt}\right)$

$\frac{s+\alpha}{(s+a)(s+b)}$	$\frac{1}{b-a}\left((\alpha-a)e^{-at}-(\alpha-b)e^{-bt}\right)$
$\frac{1}{(s+a)(s+b)(s+c)}$	$\frac{e^{-at}}{(b-a)(c-a)}+\frac{e^{-bt}}{(c-b)(a-b)}+\frac{e^{-ct}}{(a-c)(b-c)}$
$\frac{s+\alpha}{(s+a)(s+b)(s+c)}$	$\frac{(\alpha-a)e^{-at}}{(b-a)(c-a)}+\frac{(\alpha-b)e^{-bt}}{(c-b)(a-b)}+\frac{(\alpha-c)e^{-ct}}{(a-c)(b-c)}$
$\frac{\omega}{s^2+\omega^2}$	$\sin\omega t$
$\frac{s}{s^2+\omega^2}$	$\cos\omega t$
$\frac{s+\alpha}{s^2+\omega^2}$	$\frac{\sqrt{\alpha^2+\omega^2}}{\omega}\sin(\omega t+\theta)$ $\quad \phi=\tan^{-1}\left(\frac{\omega}{\alpha}\right)$
$\frac{s\sin\theta+\omega\cos\theta}{s^2+\omega^2}$	$\sin(\omega t+\theta)$
$\frac{1}{s(s^2+\omega^2)}$	$\frac{1}{\omega^2}(1-\cos\omega t)$
$\frac{s+\alpha}{s(s^2+\omega^2)}$	$\frac{\alpha}{\omega^2}-\frac{\sqrt{\alpha^2+\omega^2}}{\omega^2}\cos(\omega t+\phi)$ $\quad \phi=\tan^{-1}\frac{\omega}{\alpha}$
$\frac{1}{(s+a)(s^2+\omega^2)}$	$\frac{e^{-at}}{a^2+\omega^2}+\frac{1}{\omega\sqrt{\alpha^2+\omega^2}}\sin(\omega t-\phi)$ $\quad \phi=\tan^{-1}\frac{\omega}{a}$
$\frac{1}{(s+a)^2+b^2}$	$\frac{1}{b}e^{-at}\sin bt$
$\frac{1}{s^2+2\xi\omega_n s+\omega_n^2}$	$\frac{1}{\omega_n\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin\omega_m\sqrt{1-\xi^2}\,t$
$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos bt$
$\frac{s+\alpha}{(s+a)^2+b^2}$	$\frac{\sqrt{(\alpha-a)^2+b^2}}{b}e^{-\alpha t}\sin(bt+\phi)$ $\quad \phi=\tan^{-1}\frac{b}{\alpha-a}$

$\frac{1}{s\left[(s+a)^2+b^2\right]}$	$\frac{1}{a^2+b^2}+\frac{1}{b\sqrt{a^2+b^2}}e^{-\alpha t}\sin(bt-\phi)$ $\phi=\tan^{-1}\frac{b}{-a}$
$\frac{1}{s\left(s^2+2\xi\omega_n s+\omega_n^2\right)}$	$\frac{1}{\omega_n}-\frac{1}{\omega_n^2\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin\left(\omega_n\sqrt{1-\xi^2}\,t+\phi\right)$ $\phi=\cos^{-1}\xi$
$\frac{s+\alpha}{s\left[(s+a)^2+b^2\right]}$	$\frac{\alpha}{a^2+b^2}+\frac{1}{b}\sqrt{\frac{(\alpha-a)^2+b^2}{a^2+b^2}}e^{-\alpha t}\sin(bt+\phi)$ $\phi=\tan^{-1}\frac{b}{c-a}-\tan$
$\frac{1}{(s+c)\left[(s+a)^2+b^2\right]}$	$\frac{e^{-ct}}{(c-a)^2+b^2}+\frac{e^{-at}\sin(bt-\phi)}{b\sqrt{(c-a)^2+b^2}}$ $\phi=\tan^{-1}\frac{b}{c-a}$

Final Value Theorem for Laplace transform pairs:

$$\lim_{t\to\infty}\theta(t)=\lim_{s\to0}s\theta(s)$$

Ziegler-Nichols tuning rule based on critical gain and period

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$
$$= 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s\right)$$
$$= 0.075K_{cr}P_{cr}\frac{\left(s + \frac{4}{P_{cr}}\right)^2}{s}$$

Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Ziegler-Nichols tuning rule based on the strep response of the system:

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$
$$= 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right)$$
$$= 0.6T\frac{\left(s + \frac{1}{L}\right)^2}{s}$$

Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

Properties of the Fourier Transform

	Time-Domain	Frequency Domain
Linearity	$a_1 s_1(t) + a_2 s_2(t)$	$a_1 S_1(f) + a_2 S_2(f)$
Conjugate Symmetry	$s(t) \in \mathbb{R}$	$S(f) = \overline{S(-f)}$
Even Symmetry	$s(t) = s(-t)$	$S(f) = S(-f)$
Odd Symmetry	$s(t) = -s(-t)$	$S(f) = -S(-f)$
Scale Change	$s(at)$	$\frac{1}{\lvert a \rvert} S\left(\frac{f}{a}\right)$
Time Delay	$s(t - \tau)$	$e^{-(i2\pi f\tau)} S(f)$
Complex Modulation	$e^{i2\pi f_0 t} s(t)$	$S(f - f_0)$
Amplitude Modulation by Cosine	$s(t)\cos(2\pi f_0 t)$	$\frac{S(f-f_0)+S(f+f_0)}{2}$
Amplitude Modulation by Sine	$s(t)\sin(2\pi f_0 t)$	$\frac{S(f-f_0)-S(f+f_0)}{2i}$
Differentiation	$\frac{\mathrm{d}}{\mathrm{d}t}\ s(t)$	$i2\pi f S(f)$
Integration	$\int_{-\infty}^{t} s(\alpha)\ \mathrm{d}\ \alpha$	$\frac{1}{i2\pi f}\ S(f)$ if $S(0) = 0$
Multiplication by t	$ts(t)$	$\frac{1}{-(i2\pi)}\ \frac{\mathrm{d}S(f)}{\mathrm{d}f}$
Area	$\int_{-\infty}^{\infty} s(t)\ \mathrm{d}\ t$	$S(0)$
Value at Origin	$s(0)$	$\int_{-\infty}^{\infty} S(f)\ \mathrm{d}\ f$
Parseval's Theorem	$\int_{-\infty}^{\infty} (\lvert s(t) \rvert)^2\ \mathrm{d}\ t$	$\int_{-\infty}^{\infty} (\lvert S(f) \rvert)^2\ \mathrm{d}\ f$

Properties of Discrete time Fourier transform

Linearity	$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
Shift in Time	$x[n-k]$	$X(e^{j\omega})e^{-j\omega k}$
Shift in frequency	$x[n]e^{jan}$	$X(e^{j(w-a)})$
Time scaling	$x\left[\frac{n}{k}\right]$	$X(e^{j(k\omega)})$
Time reversal	$x[-n]$	$X(e^{-j\omega})$
Time conjugation	$x[n]^*$	$X(e^{-j\omega})^*$
Time reversal and conjugation	$x[-n]^*$	$X(e^{j\omega})^*$
Derivative in frequency	$\frac{n}{j}x[n]$	$\frac{dX(e^{j\omega})}{d\omega}$
Integral in frequency	$\frac{j}{n}x[n]$	$\int_{-\pi}^{\omega} X(e^{jv})dv$
Convolve in time	$x[n] * y[n]$	$X(e^{j\omega}).Y(e^{j\omega})$
Multiply in time	$x[n].y[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{jv}).Y(e^{j(\omega-v)})dv$